THERMAL-CRISIS EFFECT IN A SUPERSONIC FLOW CONTAINING A FIXED HEAT SOURCE

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A thermal crisis occurs in a gas flow if there is internal energy production (chemical reaction or spontaneous condensation) or in response to heat input in a given part of the flow (heating in a discharge or laser beam, etc.) [1, 2]. A source moving with the flow and producing more than a critical output gives rise to self-similar solutions to the gas-dynamic equations as detonation waves traveling along the flow [1-3]. If the source is immobile, such solutions in general do not exist, as there is no self-similarity. Nevertheless, if the initial flow is supersonic, one can construct an analytic solution for a stationary shock wave in the planar one-dimensional case, with that wave emerging from the fixed source, whose output exceeds the critical value, and which propagates upstream. Here we construct such a solution.

We neglect transients related to the shock wave (SW) formation and examine the flow from the time when the stationary SW has left the source zone. There are four flow zones (Fig. 1). The unperturbed supersonic flow travels to the left, which has parameters u_0, ρ_0, p_0, M_0 (u is speed, ρ density, p pressure, and M Mach number), and then behind the traveling SW there is a homogeneous subsonic flow having u_1, ρ_1, p_1, M_1. In the heating region, there is a stationary inhomogeneous flow having parameters u(x), ρ(x), p(x) and M(x) (x is the coordinate along the flow), while behind it there is a fresh homogeneous flow having u_2, ρ_2, p_2 and M_2.

The flow in the heating zone is stationary, so the parameters on opposite sides of it are related by the usual conservation laws:

$$\rho_1 u_1 = \rho_2 u_2, \quad p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2,$$
$$\rho_1 u_1 \left(\varepsilon_1 + \frac{u_1^2}{2}\right) + p_1 u_1 = \rho_2 u_2 \left(\varepsilon_2 + \frac{u_2^2}{2}\right) + p_2 u_2 + w. \tag{1}$$

Here $\varepsilon = c^2/\gamma(\gamma - 1)$ is the specific internal energy, c the speed of sound, and γ the adiabatic parameter. The detailed form of the w term in the third equation in (1) is dependent on the source type. On bulk heating, w = Q (Q is the total amount of heat absorbed in unit time in unit volume in the source region). For a mass source, w = ρuq (q is the total specific heat uptake per unit mass of the flow in the same part).

The flow parameters on the two sides of the SW are related by the standard formulas for a shock wave, which can be taken [4] as

$$\rho_1(u_1 - D) = \rho_0(u_0 - D), \; p_1 + \rho_1(u_1 - D)^2 = p_0 + \rho_0(u_0 - D)^2,$$
$$\varepsilon_1 + \frac{p_1}{\rho_1} + \frac{1}{2}(u_1 - D)^2 = \varepsilon_0 + \frac{p_0}{\rho_0} + \frac{1}{2}(u_0 - D)^2, \tag{2}$$

where D = dx/dt is the SW speed.

As (2) contains the additional parameter D, (1) and (2) are insufficient to define the flow parameters at the exit from the heating region via the parameters of the incident flow and the given heat production. However, numerical calculations show that the following major point can be utilized. Directly behind the SW, the flow is subsonic, so stability in that

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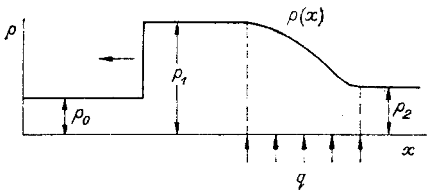

Fig. 1

state as a whole is provided only by the condition that the Mach number is strictly one at the exit from the heating source zone (the analog of the Jouguet condition in detonation-wave theory):

$$M_2 = u_2/c_2 = 1. \tag{3}$$

In fact, for any smaller M_2, the stationary state in the source region would be disrupted by perturbations traveling upstream from the right-hand boundary, whereas if it were larger, such a stationary state would be impossible even in principle [1, 2].

When (3) is obeyed, the total heat production behind the SW is precisely critical and then one can write [2] for a mass heat source that

$$q = \frac{c_1^2}{2(\gamma^2-1)}\left(M_1 - \frac{1}{M_1}\right)^2 \tag{4}$$

(for a bulk heat source it is sufficient to remember that $q = Q/\rho_1 u_1$).

We introduce the dimensionless parameter $\lambda = (u_0 - D)/c_0$ and get from (2) that

$$u_1 = c_0\left[M_0 + (1-\mu^2)\left(\frac{1}{\lambda} - \lambda\right)\right],\quad c_1^2 = c_0^2(1-\mu^2+\mu^2\lambda^2)\left(1+\mu^2-\frac{\mu^2}{\lambda^2}\right),$$
$$\frac{1}{\rho_1} = \frac{1}{\rho_0}\left[(1-\mu^2)\frac{1}{\lambda^2} - \mu^2\right],\quad p_1 = p_0[(1+\mu^2)\lambda^2 - \mu^2]\ (\mu^2 = (\gamma-1)/(\gamma+1)). \tag{5}$$

We first consider the simpler case of a mass source, and by $q_* = \frac{c_0^2}{2(\gamma^2-1)}\left(M_0 - \frac{1}{M_0}\right)^2$ we denote the critical heat production for the incident supersonic flow, and introduce the transcritical parameter $\alpha = q/q_* - 1$; then (4) gives

$$(1+\alpha_1)\left(M_0 - \frac{1}{M_0}\right)\frac{u_1}{c_0} - \frac{c_1^2 - u_1^2}{c_0^2} = 0 \quad (1+\alpha_1 = \sqrt{1+\alpha}). \tag{6}$$

Then (5) and (6) give a closed equation of fourth degree in λ:

$$(1+\alpha_1)\left(M_0 - \frac{1}{M_0}\right)\left[M_0 + (1-\mu^2)\left(\frac{1}{\lambda} - \lambda\right)\right] + M_0^2 + 4\mu^2 - 3 +$$
$$+ 2M_0(1-\mu^2)\left(\frac{1}{\lambda} - \lambda\right) + \lambda^2(1-3\mu^2) + \frac{1-\mu^2}{\lambda^2} = 0. \tag{7}$$

As the SW moves in the opposite direction to the flow ($D \le 0$), $\lambda \geqslant M_0$, and physical meaning attaches only to any flow having a positive direction for the velocity in the source zone ($u_1 \geqslant 0$). (5) implies that $\lambda < \lambda_*$ ($\lambda_* = M_0/2(1-\mu^2) + [M_0^2/4(1-\mu^2) + 1]^{1/2}$), and it can readily be shown that there is a single root to (7) in the segment (M_0, λ_*) for $M_0 > 1$.

Similarly, we get a closed equation of eighth order in λ for a bulk source:

$$(1+\alpha)\left(M_0 - \frac{1}{M_0}\right)^2 M_0\left[(1-\mu^2)\frac{1}{\lambda^2} + \mu^2\right]\left[M_0 + (1-\mu^2)\left(\frac{1}{\lambda} - \lambda\right)\right] -$$
$$-\left[M_0^2 + 4\mu^2 - 3 + 2M_0(1-\mu^2)\left(\frac{1}{\lambda} - \lambda\right) + \lambda^2(1-3\mu^2) +\right.$$
$$\left. + \frac{1-\mu^2}{\lambda^2}\right]^2 = 0 \quad (\alpha = Q/Q_* - 1). \tag{8}$$

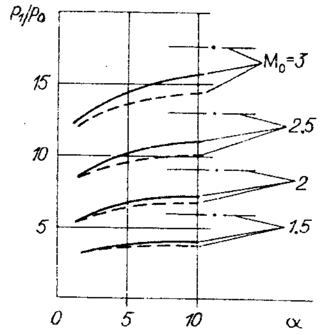

Fig. 2

As in the previous case, we can show that the (M_0, λ_*) segment in (8) has a single root for $M_0 > 1$. Therefore, the root in (7) or (8) is localized and there is no difficulty in finding λ in each of the cases.

Figure 2 shows the α dependence of the pressure increase p_1/p_0. The solid lines correspond to a bulk source and the dashed ones to a mass one. The horizontal lines denote the levels at which the curves flatten off for infinite heat production.

The results obtained directly from the solution agree very well with numerical calculations from the nonstationary gas-dynamic equations.

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SHEAR INTERFEROMETRY APPLIED TO THE DENSITY DISTRIBUTION IN A LAMINAR BOUNDARY LAYER

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The theoretical models used for laminar supersonic boundary layers have not been tested by experiment because of the limitations in classical measurement methods. Pneumometric methods employ sensors whose sizes are comparable with the boundary-layer thickness (that thickness is not more than 1 mm in supersonic wind tunnels). Thermoanemometers give good resolution but the characteristics alter in the transonic region, which makes them difficult to use. Also, the sensor may distort the boundary layer.

Indirect methods are used in measuring density, temperature, and velocity profiles for supersonic flows; these methods require additional assumptions and empirical constants to give quantitative results. Optical methods enable one to measure density profiles without perturbing the flow. The spatial resolution in an interference method is restricted to $d = 1.6\sqrt{\lambda L}$ (λ wavelength and L size along the beam), which means that a flat object 200 mm wide allows one to obtain independent readings with a step of 0.5 mm [1]. The resolution can be increased by one or two orders of magnitude on a cylindrical object, but then the total phase shift along the beam in the laminar layer becomes comparable with the usual sensitivity of 0.05-0.1 λ. The sensitivity is limited by the accuracy in determining the turning points,

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